A Tool or A Collaborator? Rethinking Mathematical Intuition in the Age of AI

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Abstract

The integration of artificial intelligence (AI) into mathematical practice raises profound questions about the nature of mathematical intuition in discovery and research. This paper examines the epistemological shifts induced by AI-based programs and tools, and explores how they challenge traditional notions of intuition. Building on four case studies (AlphaGeometry, knot theory, FunSearch of cap sets, and the bunkbed conjecture), we argue that AI-based systems do not abolish intuition but relocate it, shifting it from a primarily anticipatory capacity to one that can also operate retroactively as interpretation, reconstruction, critique, and integration. Drawing on the historical flexibility of the concept, we propose a working definition of mathematical intuition as a family of practice-embedded capacities for orienting, evaluating, and rendering intelligible mathematical moves and outcomes across hybrid human–machine workflows. Ultimately, we argue that AI-based programs, even when their capacities are more limited than some narratives suggest, are not merely new computational aids but a transformative epistemic force that reconfigures what it means to "do mathematics" in the 21st century.

Keywords: Mathematical Intuition; AI in Mathematics; Mathematical Practice; Human-Machine Collaboration; Mathematical Reasoning

1. Introduction

Artificial intelligence (AI) is rapidly transforming multiple domains, and mathematics is no exception. Traditionally viewed as the pinnacle of human intellectual achievement, defined by rigorous proofs and deep theoretical insights, mathematics is now facing the possibility of fundamental change with the advent of AI-based programs that promise to solve complex problems and even generate formal proofs. The question at hand is no longer whether such technologies can assist with mathematical tasks, but whether these can fundamentally alter the nature of mathematical practice itself.

Here, we explore the philosophical implications of these AI-based technologies in mathematics – such as *AlphaGeometry* or *FunSearch*, focusing on how they challenge the concept of

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mathematical intuition. Mathematical intuition - usually considered the 'mark' of *human* reasoning - is being challenged by today's AI-based technologies that are capable of solving complex problems at the International Mathematical Olympiad, achieving silver- or gold-medal standards, and creating "new knowledge" (Romera-Paredes et al., 2024, p. 473). Recent developments suggest a notable expansion of AI-assisted proof capabilities: *AlphaProof* has been presented as able to generate and search for proofs in *Lean*, while using successful proof attempts as feedback to improve performance on more challenging problems. *AlphaGeometry2* has likewise been reported to solve approximately 83% of the geometry problems drawn from past IMO competition problem sets spanning roughly the last 25 years.

As AI-based technologies continue to encroach on areas once thought to be exclusively human, we question whether the unique human qualities that have historically characterized mathematical developments will remain characteristics of mathematical activity, or whether they will be supplemented or augmented in new ways by these new technologies. We suggest that AI-based programs' ability to explore vast problem spaces and find connections (even if merely statistical ones) or programs that humans may overlook (or cannot even find) suggests that these technologies do more than offer merely "brute-forcing" solutions or passive calculating power. These programs are actively engaged in the process of creating knowledge, by performing operations that are not only quantitatively but also – and this is our claim here – qualitatively not available to the human mathematician; these processes hence could be seen as prompting a new form of heuristic reasoning, and hence call to rethink mathematical intuition.

While various papers have already suggested rethinking mathematical practices in light of these developments (for example Kantamneni & Tegmark, 2025; Gastaldi, 2024; Pantsar, 2024; Testolin, 2024; Davis, 2023), we focus on the notion of mathematical intuition. Intuition is fundamental to a wide range of mathematical practices and concepts, including creativity, agency, understanding, and reasoning, and it functions as a central bridging notion between the informal, experience-based aspects of mathematical work and the formal products that ultimately count as proofs, definitions, and theorems. Precisely because intuition is routinely invoked to explain how mathematicians select promising paths, recognize structure, and judge what is worth pursuing, it offers a particularly revealing lens for diagnosing what is changing when AI-based technologies begin to participate in discovery and proof production. We therefore foreground intuition by attending to its flexibility as a concept and by examining its historical versatility. Such a historical examination shows that these issues are not entirely new, since they echo debates going back at least to the 17th century. What is new - and this is one of the main claims of this paper - is the place assigned to the mathematician's intuition within the emerging constellations of AI-based mathematical practice.

The paper begins in Section 2 with a brief and selective historical review of machines in mathematics, in order to establish a baseline picture of machines as aids and to foreground earlier tensions between mechanical procedure and intuition. Section 3 then traces the historical plasticity of mathematical intuition and situates contemporary usage of the term as a plural, practice-sensitive concept that is nevertheless often treated as structurally prior to proof and discovery. Building on this conceptual groundwork, Section 4 examines four recent AI-assisted mathematical episodes (AlphaGeometry, knot theory, FunSearch on cap sets, and the bunkbed conjecture) to show how different human–machine configurations reorganize mathematical intuition. Section 5 synthesizes these case studies and argues that AI does not abolish intuition but relocates it, expanding it from an exclusively anticipatory faculty into a set of hybrid practices. Finally, Section 6 draws out the broader philosophical implications for mathematical practice and briefly indicates open problems and limits that remain.

2. Mathematics as Aided by Machines

The idea that mathematics might be relegated to an automatic machine appeared long before Turing's 1948 paper, "Intelligent Machinery" (Turing, 2004 [1948]). In that text, Turing discusses the possibility of what we might call "thinking machines," an idea already mentioned by Leibniz, who spoke of "an arithmetical instrument, which carries over all the work of the mind to wheels."[2] Leibniz's stepped reckoner was one of the first examples of an automated machine, far beyond the technology of its time, and it was the first calculator that could perform all four basic arithmetic operations. Leibniz's notion of transferring "the work of the mind to wheels" may also refer to Blaise Pascal's Pascaline, another mechanical calculator invented, designed, and built by Pascal between 1642 and 1644. The Pascaline had three versions, one for accounting, one for surveying (measurement), and one for science (decimal calculation). Both Pascal's and Leibniz's machines aimed to fulfill the dream of a powerful calculator that would save mathematicians from their "slavery." In other words, the mechanical calculator was explicitly understood as a tool for taking the burden of calculation off the mathematician.

Moving past the 17th century, Charles Babbage's Analytical Engine, developed in the 1830s, represented a conceptual leap toward the ideal of universal mechanical computation. Unlike earlier calculating machines, it possessed both a "memory" and a "processor," controlled by punched cards that could execute predetermined sequences of operations. Another step in the mechanization of mathematics was introduced by William Stanley Jevons' Logical Piano, developed during the 1860s, which attempted to mechanize logical reasoning itself, not merely arithmetic calculation. By reducing syllogistic logic to mechanical operations using levers and keys resembling a piano, it suggested that formal logical deduction could be performed automatically, thereby prompting a new wave of thinking about deduction as potentially mechanical. These new technologies, however, were not accepted without criticism. Henri Poincaré criticized such purely mechanical approaches to mathematics by comparing Jevons' piano to Chicago meat-processing machines, thereby highlighting the tension between formal mechanical procedures and mathematical intuition, and insisting that "only intuition could enable us to distinguish among" logical rules (Poincaré, 1913, p. 473)[3].

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[2] (Leibniz 1923-, series VII, vol. 6, p. 487–488): "Qui Geometrico sunt ingenio eorum inventa difficilia sunt, et profunda, et multa meditatione expressa. Qualia nec facile enuntiantur, nec statim a quovis auditore aut spectatore intelliguntur, unde Exemplum elegans habemus in Machina textrice, nunc passim frequentata, Scoti cuiusdam invento, quod novennio integro occupavit autorem suum, aut in Arithmetico instrumento, quod omnem animi laborem in rotas transfert."

[3] According to Poincaré: "to demonstrate a theorem, it is neither necessary nor even advantageous to know what it means. The geometer might be replaced by the logic piano imagined by Stanley Jevons; or, if you choose, a machine might be imagined where the assumptions were put in at one end, while the theorems came out at the end, like the legendary Chicago machine where the pigs go in alive and come out transformed into hams and sausages" (1914 [1908], p. 147). However he notes that he does "not wish to say that mathematics could be created by an utterly unintelligent being [but] the role of intelligence is limited to choosing from a limited arsenal of rules given in advance" (1905, p. 825).

The beginning of the 20th century saw modern logic show that various mathematical domains (geometry, set theory, and arithmetic) can be captured within formal axiomatic systems[4], which helped motivate the idea of mechanizing mathematical procedures, including proofs. In the 1950s, computer-based verifications of a finite number of cases for general theorems were taken to provide a positive indication that the theorem is correct. Such computer-assisted calculations could supply evidence of correctness, but they were not yet considered a proof[5]. That changed in the 1970s, when the computer-assisted proof of the four-color theorem was presented. The four-color theorem, stating that no more than four colors are required to color the regions of any map so that no two adjacent regions have the same color, was the first major theorem to be proved with extensive computer assistance, and as such it generated extensive discussion. At first, it was not accepted by the mathematical community, and it raised philosophical objections regarding whether it should even count as a proof (Parshina 2023). The major objection was that it was unverifiable, since it would take over 1000 hours for a human to check by hand all of its 1,834 reducible configurations. Eventually it was accepted by the community after several rounds of revision and the identification of errors[6]; yet the proof remained unverifiable. One may argue that this concern still applies to the use of AI-based technologies[7] such as AlphaProof or AlphaGeometry, as they produce very long proofs that sometimes cannot be verified. Against this background, what is the difference between the four-color theorem proof and AI-generated proofs?

One striking difference is that, in contrast to Leibniz's suggestion, the machine (or AI-based technologies, for our purposes) is no longer merely a **passive** device that can only calculate or automate manual procedures. AI-based proofs are **active** in the sense that they provide new, meaningful outcomes that mathematicians are not necessarily able to come up with on their own. This shift is already foreshadowed by the proof of the four-color theorem: the mathematician herself was no longer considered (by various philosophers of mathematics) to be the sole author of the mathematical proof or text, since other, non-human actors take part in the proof of such theorems[8]. This raises a practical and philosophical question about the role of the mathematician in contemporary proving practices[9]. Where does that position the working mathematician in the proving process? Is she guiding the process, is her intuition being guided by the results provided by the machine, or is she following the machine's own intuition?

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[4] For example, Zermelo's set theory (1908) and Russell and Whitehead's Principia Mathematica (1910)

[5] This was indeed the case for the various verifications that Fermat's last theorem: in 1954 it was verified with the computer that for the first 2,000 integers n for $n > 2$ the equation $a^n + b^n = c^n$ does not have a solution in positive integers (see: Lehmer et al. 1954).

[6] The appearance of *Every Planar Map is Four-Colorable*, Appel and Haken's book claiming a complete and detailed proof (with a microfiche supplement of over 400 pages) marked the acceptance of this episode, including their response to the rumors of flaws in their proof (Appel and Haken 1989).

[7] Concerning the non-surveyability, see: (Berg & Tanswell, 2025) as well as section 4 below.

[8] Compare here Hiller (2023, p. 284), whose insights on AI- and machine-writing can be easily transferred to the domain of mathematical writing: It is "crucial to abandon the double standard assumption of all writing - texts are always of human origin and the devices of passive instrumentality [...]."

[9] Such a shift in the role of the mathematician is seen more clearly with programs such as Lean and other automated proof-assistants, as Coq, which were developed during the 2010s (Avigad 2024b; for the history of the attempts to mechanize proofs in the 20th century, see also MacKenzie 2004). This shift points to a more general shift in the practices of mathematics in recent years. That is, the formalization and automation done by such technologies evoke Turing's early vision of "thinking machines" capable of mathematical reasoning, but nevertheless, one cannot call these technologies 'generative', but merely 'symbolic AI' (Avigad, 2024a).

To address these questions, we first need to clarify two concepts: the "role" of the mathematician (or the "working mathematician") and mathematical intuition. The term "working mathematician" is often used to characterize the mathematician as a practitioner actively engaged in research, focusing on applicable methods rather than purely abstract theorization or philosophical considerations (Bourbaki 1949; Mac Lane 1971; Corry 2004). Within this characterization, the mathematician's role was to prove and theorize, to discover intuition or be guided by it, and to produce new knowledge, whether in the form of new, more aesthetic proofs or completely new proofs. The question at stake is what changes when AI tools enter the scene, with their ability to verify and create proofs, come up with connections between elements for new theories, search huge datasets and develop "intuition," produce new knowledge, and verify and write papers. To understand where this positions the human mathematician, and what it entails for intuition which is often treated as an entirely human capacity, we need to examine the concept of intuition more carefully and trace the developments it has undergone.

3. Mathematical Intuition: A flexible and evolving concept

First a word of clarification is in order: we are not going to present here a full-blown historical survey of the notion of intuition in mathematics, nor all of the historical narratives related to it[10]. We intentionally omit these histories not because they are not important but because our goal here is to focus on the historical flexibility of the notion, in order to understand its potential future flexibilities with the rise of AI. Given this caveat, we shall begin with Kant, since he was the one who (re)shaped such a notion of intuition – in the sense of *Anschauung* – in the Western world during the 19th and the 20th centuries. Kant's views on the pure form of intuition (*Anschauung*) and its relation to mathematics, and specifically regarding pure space as a formal condition for any sensory intuitions, made an enormous impact on the mathematical community (Janiak 2022). Such a conception necessarily tied geometry as well as geometrical concepts with the spatiotemporal ability of human beings to recognize objects with their senses. However, the discovery of non-Euclidean geometries during the 1830s complicated this understanding of intuition: not only that 'the' space as such no longer 'naturally' existed or was given, but also establishing the consistency of these newly discovered (as well as Euclidean) geometries became unclear and debatable. This led to a rethinking of the status of spatial mathematical intuition (Epple, 2022).

The discovery of projective geometry, higher-dimensional spaces, and 'monstrous' functions also undermined the status of mathematical intuition (Volkert 1986; Volkert 1987; Poincaré 1899; 1914; Borel 1922), leading directly into the early 20th-century debates about how intuition should be constrained and redefined, and accumulating in Hans Hahn's "crisis of intuition" speech (Hahn, 1980 [1933]). One approach to deal with the changes of the status of intuition was to turn to methods aligned with mathematical formalism (either in its extreme form, considering

[10] Appeals to "intuition" in connection with mathematical cognition and practice appear across multiple eras, bodies of scholarship, and intellectual traditions, each bringing its own meanings and connotations. Limiting ourselves to a few especially influential reference points, one might mention Brouwer's philosophy of mathematics, Gödel's epistemological reflections, Fischbein's research programme in mathematics education, Kahneman's System 1/System 2 framework, Gigerenzer's work on heuristics in psychology, and the informal ways contemporary mathematicians speak about intuition in practice. None of them will be surveyed here, both for reasons of space and because doing so would fall outside our present focus.

'mathematics as a game of symbols'[11] or in its less strict versions[12]); another approach emphasized intuition as referring only to palpable visuality (*Anschaulichkeit*), highlighting the plasticity of mathematical intuition: Felix Klein, for example, introduced a distinction between 'naïve' and 'refined' intuition and even between various forms of intuition (Klein 1911). Another example is the Italian mathematician Federigo Enriques, who described different forms of intuition, including the intuition of "what can be seen" (Enriques 1938, pp. 173–174),[13] underlying that intuition is guiding mathematical practices (De Toffoli & Fontanari 2022).

The late 20th century repositioned intuition as distinctly human, particularly following the reactions to computer-assisted proofs and visualizations, and echoing developments in the philosophy of mathematical practice. The emerging computer-assisted proofs, the personal computer and computer based visualizations have changed how one considers intuition. Following the objections raised against the computer-assisted proofs during the 1970s and later[14] intuition was reposited as a mark of the humane; with the rise of the software and personal computers in the 1980s, intuition was considered "as a lived, embodied, and sensory capacity which distinguishes humans from machines" (Pedwell 2024, pp. 197-8). Moreover, the 1970s witnessed the revolutionary work of the mathematician Benoit Mandelbrot, using computer graphics to visualize fractals, demonstrating how computational power could reveal mathematical structures previously invisible to the human eye (Samuel 2020). Mandelbrot was not alone in this endeavor: in the 1960s Alan H. Schoen employed FORTRAN and computer graphics programs to generate videos of periodic surfaces at the MIT Lincoln Laboratory.[15] These examples highlight that intuition was regarded as a cognitive property unique to the human being. The emergence of philosophy of mathematical practice as its own field of research gave rise to the view that mathematical intuition can be considered as a form of experience akin to sensory perception and visual reasoning (de Toffoli 2021; Giardino 2018), and at the same time may be directed toward abstract objects rather than concrete, material ones (Chudnoff 2011; 2014).

Even this brief and selective historical overview is enough to make one point clear: mathematical intuition has never been a fixed notion with a stable referent. Instead, it has repeatedly been re-specified and redistributed across different mathematical projects, epistemic ideals, and technological contexts, which is precisely what makes it a useful lens for thinking about its possible reconfiguration under AI-based technologies. With that historical plasticity in view, it becomes easier to situate current uses of the term, which continue this pattern of flexibility.

Recent discussions inherit this late–twentieth-century repositioning of intuition, but they frame it in a more explicitly pluralist and practice-oriented way. Rather than treating mathematical intuition as a single, unified faculty, recent literature increasingly understands it as a family of capacities

[11] The turn to formalism represents a rejection of intuition by prioritizing a system of rules and symbol manipulation, essentially viewing mathematics as a game played with symbols rather than a reflection of inherent truths accessible through intuition. As a result, mathematical truths are derived solely from the established axioms and rules, not from any inherent understanding of the concepts involved (Frege, 1903; Putnam & Benacerraf 1984; Detlefsen, 1993; 2005).

[12] Compare here Mehrtens (1990), who describes the differentiation between axiomatic-based approaches to mathematics and intuition-based approaches within the framework of the opposition between 'modernism' and 'counter-modernism'.

[13] See also: (Bussotti & Pisano 2015).

[14] See above the discussion above on the 4-color theorem

[15] See: https://schoengeometry.com (last accessed on: 15 September 2025).

that operate within mathematical work and help organize inquiry. On this view, intuition includes heuristic plausibility judgments and model-based anticipation that guide mathematicians in selecting what is worth pursuing and in projecting which directions are likely to be fruitful (Hersh 2011). In the view of "mature intuition", it is characterized as fast, automatics, fluent, reliable, and insightful domain inference typical of expert performance, where insightful is taken as pointing the way toward substantive, nonobvious truths (D'Alessandro & Stevens 2024; Dehaene 2009). Mathematics education research supports this pluralist orientation by emphasizing the need to classify the many manifestations of mathematical intuition rather than forcing them into a single narrow definition (Lajos 2023). At the same time, analytic philosophy continues to develop an experience-based characterization, on which mathematical intuition is modeled as a perception-like experience that can present mathematical content to the subject (Chudnoff 2019; 2020). Taken together, these strands suggest a disciplined pluralism: mathematical intuition functions today as an umbrella term for several epistemically and methodologically distinct roles within mathematical practice, and many of these roles preserve an anticipatory dimension, even when intuition is treated as trainable and historically variable rather than as a fixed psychological kind.

On these contemporary accounts, intuition is not only associated with human experience and expertise; it is also often assigned a structurally prior role in mathematical work. It is described as an insight that, even when cultivated and refined, typically comes before the proof and helps orient problem-solving, subsequent formalization, and pointing towards the 'truth', rather than being produced by deduction after the fact[16]. The emergence of AI-based tools therefore sharpens two related questions: first, whether "intuition" can still be reserved for human cognition alone, and second, whether intuition should still be understood primarily as anticipatory. We will address the first question through a careful examination of the second question using four examples of AI-generated mathematics in the next section.

4. Intuition as Anticipatory, Retroactive, Hybrid and Co-constitutive

Unlike its historical casting as preceding formal reasoning, in this section we aim to show how AI-based tools challenge the anticipatory orientation of mathematical intuition by producing results that lack prior human anticipation, forcing intuition to operate retroactively as interpretation, assimilation, or critique. It is important to note that this retroactive stance does not appear uniformly; the various cases reviewed below reveal different (temporal) modes of co-constitution of intuition between human and machine, each with a distinct temporal and epistemic structure. For each case, we therefore analyze two axes: whether intuition functions anticipatorily or retroactively, and whether it remains exclusively human or becomes co-constitutive within a hybrid human–machine practice.

In what follows, we examine four recent mathematical discoveries done with AI-based technologies, and in doing so, we inspect how such discoveries are presented by the actors themselves. We chose to focus on these specific examples because some come from domains that rely heavily on geometric intuition or visual reasoning (such as Euclidean geometry, knot theory, and graph theory) while others challenge or even disprove conjectures traditionally regarded as 'intuitive'. The first three examples have been extensively discussed in research papers as they

[16] This anticipatory view is noted by Poincaré in 1908, in his *Science and Method* (1914 [1908], p. 129): for "the pure geometrician himself this faculty [intuition] is necessary: it is by logic that we prove, but by intuition that we discover."

illustrate different types of achievements made possible by AI-based technologies,[17] while the fourth highlights their limitations and has received less attention. However, the current literature on these examples is mainly descriptive and offers no critical point of view, let alone any attempt of a philosophical discussion on the nature of such examples. This literature also does not attempt to frame the longer historical and philosophical traditions and trajectories which stand at the backdrop of these developments. We present the examples with a focus on how they have been interpreted by different scholars, and then discuss their philosophical implications. The selection of examples is certainly not exhaustive, and additional cases could be included[18].

4.1 Retroactive and Co-constitutive: The case of AlphaGeometry

We begin with one of the most widely discussed recent examples: solving problems from the International Mathematical Olympiad (IMO). Trinh et al. (2024) report that these AI-based systems reach what they describe as a "silver-medal standard,"[19] and in some cases approach the level of an average IMO gold medallist. The problems in question were solved using AlphaProof, AlphaGeometry, and AlphaGeometry2. A central point in their account is that these systems can generate the auxiliary constructions needed for a solution on their own, rather than relying on human-provided demonstrations (Trinh et al. 2024, p. 478). In their paper, Trinh et al. also highlight a tension between the strengths of formal and natural-language mathematics: formal languages have the advantage that each inference step can be checked for correctness, but the available stock of human-written formal data remains limited. AlphaGeometry developers team therefore frames their aim as bridging these two "complementary spheres" by developing a model that translates natural-language problem statements into formal statements, thereby connecting human mathematical expression with formally verifiable reasoning[20] .

Once a problem is given, AlphaProof generates candidate solutions, which are then proved or disproved; successful proofs are used to further train, or "reinforce," AlphaProof's language model. Trinh et al. report that this feedback procedure was carried out across "millions of problems", and note an early result from AlphaGeometry: in one case the system identified an "unused premise," and thereby arrived at a more general version of the translated IMO theorem itself (Trinh et al. 2024, p. 480). In another Euclidean plane geometry example[21], the system produced a solution within 19 seconds of receiving the formalized problem, an extremely short time by human standards. This naturally raises the question of whether such systems merely speed up existing practices or also change them in kind. One indication of a qualitative shift comes from

[17] See especially the chapter "AI for Mathematics", in: Miao & Wang (2024)

[18] For example, another discovery made by AI-based technologies concerns matrix multiplication: Fawzi et al (2022) used *AlphaTensor* to discover new, more efficient algorithms for multiplying 5 × 5-matrices over Z/2Z.

[19] https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/?utm_source=x&utm_medium=social&utm_campaign=&utm_content= (last accessed on: 15 September 2025)

[20] Ibid.

[21] The problem consisted only of concepts and constructions that any person in a high-school level can already understand. It is as follows: "Let ABC be a triangle with AB < AC < BC. Let the incentre and incircle of triangle ABC be I and ω, respectively. Let X be the point on line BC different from C such that the line through X parallel to AC is tangent to ω. Similarly, let Y be the point on line BC different from B such that the line through Y parallel to AB is tangent to ω. Let AI intersect the circumcircle of triangle ABC again at P ≠ A. Let K and L be the midpoints of AC and AB, respectively. Prove that KIL+YPX=180." The problem is solved by an auxiliary point construction and consequently the construction of two pairs of similar triangles.

Timothy Gowers' remark that one of the solutions involved a "non-obvious construction,"[22] suggesting that these tools may alter not only the pace but also the character of mathematical problem-solving.

Nonetheless, it is important to emphasize that AlphaGeometry does not solve every problem in plane Euclidean geometry. Trinh et al. stress that the system can solve many geometry problems without human demonstrations, but their discussion also makes its limitations visible. In particular, they note that geometry-specific languages are narrowly defined and therefore cannot express many human proofs that draw on tools outside standard Euclidean geometry, such as complex numbers, which appear in at least one human solution to an IMO problem (Trinh et al. 2024, p. 476). They also describe some solutions as having a "much less intuitive character"[23], which immediately raises the issue of whose intuition is at stake and by what standards such "intuitiveness" is being assessed.

A complementary account by Miao and Wang (2024) helps specify what AlphaGeometry relies on. They describe three main components: (1) a symbolic deduction engine that efficiently derives new statements from premises using geometric rules; (2) an algebraic component that deduces new statements using algebraic rules; and (3) what they call the "dependency difference" concept, which supports the introduction of auxiliary constructions (such as new points) in ways that exceed the capabilities of symbolic deduction engines (Miao and Wang 2024, pp. 32–33). On their description, the mathematician's role is concentrated in the "pretraining and fine-tuning of language models," and the system's outputs still require assessment by human experts. Similarly, the developers of AlphaGeometry 2 suggest that mathematicians are often needed to provide a retroactive account of the "intuitions behind the auxiliary construction."[24] On this picture, intuition is no longer primarily an anticipatory insight that precedes and enables problem-solving and formalization; it becomes something articulated after the fact, as part of making sense of what the system has produced.

To conclude, AlphaGeometry's auxiliary constructions illustrate a form of retroactive intuition by producing "non-obvious" constructions that mathematicians must later interpret and render intelligible. However, this mode of operation is different from the 19th and 20th centuries conceptions of diagrammatical reasoning, where such type of reasoning has been usually considered as an essential part of the practices of Euclidean geometry (even if sometimes illustrative). In this case, *AlphaGeometry* works only with texts, that is, with formalized statements. Any kind of visual reasoning or diagram is just nonexistent and in this sense, visual

[22] https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/ (last accessed on: 15 September 2025)

[23] For other problems, and their comparison with human solutions, see (Trinh et al. 2024). For one of the problems presented in the article, Trinh et al. note that "the human solution uses complex numbers. With a well-chosen coordinate system, the problem is greatly simplified and a solution follows naturally through algebraic manipulation. […] AlphaGeometry solution involves two auxiliary constructions and more than 100 deduction steps, with many low-level steps that are extremely tedious to a human reader. This is a case in which the search-based solution is much less readable and *much less intuitive* than coordinate bashing." (Trinh et al. 2024: under "Extended Data Fig. 3"; cursive by this paper's authors). For another problem, which could *not* have been solved by AlphaGeometry, the "human proof uses four auxiliary constructions […] and high-level theorems such as the Pitot theorem and the notion of homothety. These high-level concepts are not available to our current version of the symbolic deduction engine both during synthetic data generation and proof search. Supplying AlphaGeometry with the auxiliary constructions used in this human proof also does not yield any solution." (ibid.: "Extended Data Fig. 5")

[24] https://storage.googleapis.com/deepmind-media/DeepMind.com/Blog/imo-2024-solutions/P4/index.html (last accessed on: 15 September 2025)

intuition is not first guiding discovery or proof, but re-entering only *later*, after the proof is presented, to reconstruct meaning and motivation for the proposed construction. This can be seen as a perceptualist model of intuition (Chudnoff 2014; 2020; Van-Quynh 2017; 2019; Dehaene 2009; Cooney 1991) except the "perception" now comes secondhand, mediated by machine outputs. Philosophically, this mode exemplifies interpretive co-constitution in retrospective: the machine generates first, the human interprets later.

4.2 Anticipatory and Co-constitutive: The case of Knot theory

The second example moves from Euclidean Geometry to knot theory. In a paper with the highly promising (and somewhat ambitious) title "Advancing mathematics by guiding human intuition with AI" (Davies et al., 2021), Alex Davies and his colleagues address the connection between two branches of knot theory by finding relations between their invariants, that is, between the *signature* of a knot (a classical topological-algebraic invariant) and an invariant in hyperbolic knot theory, being the hyperbolic *volume* of a knot - a geometric invariant. To do this, they use a neural network to predict the signature based on hyperbolic invariants. As a result, the model achieves high accuracy in predicting the signature, suggesting a connection between the two properties. To gain further insight into this relation, the authors use gradient-based attribution methods to determine which hyperbolic invariants are most influential in predicting the signature. They find that the prediction depends primarily on three invariants (out of numerous invariants initially checked)[25] related to the geometric feature of the knot called the 'cusp'. This observation leads them to define a new quantity called the 'natural slope' and make an initial conjecture. After further investigation, they revised their conjecture, leading them to the proof of a relationship between *slope*(K), signature σ(K), volume *vol*(K) and one of the next most salient geometric invariants, the injectivity radius *inj*(K) (Davies et al., 2021, p. 72). The sheer amount of initial data feeded into the neuronal network must be emphasized: the authors produced a sample set of more than 2.7 million knots in order to predict the signature from a list of hyperbolic invariants.

Whether these results should be attributed primarily to AI-based guidance of human intuition is not straightforward, and has already been debated on the grounds that similar discoveries might, in principle, have been reached with earlier and less sophisticated technologies (Davis 2021). Miao and Wang (2024, p. 23) nevertheless emphasize that the mathematician remains the key agent in the process, since he is the one who "guides the selection of conjectures that not only fit the data but also look interesting." Even so, given the sheer volume of data that must be processed, it is hard to view the technology as merely a passive instrument; at minimum, it functions as a partner in generating and filtering candidate patterns.

In this example, AI detects correlations across massive datasets, while the mathematician determines which conjectures are worth exploring. While the mathematician's role is of importance here (as her intuition), we still see a shift in the concept of intuition - similar to the AlphaGeometry example - since various knot theory arguments were historically based on human

[25] These being "algebraic and geometric knot invariants. Different datasets involved computing different subsets of these […] invariants: signature, slope, volume, meridional translation, longitudinal translation, injectivity radius, positivity, Chern–Simons invariant, symmetry group, hyperbolic torsion, hyperbolic adjoint torsion, invariant trace field, normal boundary slopes and length spectrum including the linking numbers of the short geodesics". Moreover, the "three invariants [are] the real and imaginary parts of the meridional translation μ and the longitudinal translation λ". (Davies et al., 2021, p. 75)

visual reasoning (Epple 2013), and are now absent from the suggestions made by AI-based programs. Still, unlike the AlphaGeometry case, intuition here largely retains an anticipatory role: mathematicians continue to shape the space of relevance and decide which machine-generated proposals have epistemic promise. In that sense, the interaction can be described as co-constitutional: the machine provides a menu of patterns, and the human's intuition curates what becomes mathematically meaningful.

4.3 Hybrid Intuition: The case of Lower bounds of cap sets

The third example turns to the area of combinatorics. A *cap set* is defined as a subset of the n-dimensional affine space over the three-element field, where no three elements sum to the zero vector. The *cap set problem* is the problem of finding the size of the largest possible cap set, as a function of n. Determining whether a cap set exists from a collection of given elements is known to be NP-complete (yet it is known how to quickly obtain cap sets of size 2^n). In 1970, Giuseppe Pellegrino (Pellegrino 1970) proved that four-dimensional cap sets have a maximum size 20 (note that $2^4 = 16$). This result led to finding lower bounds larger than 2^n for any higher dimension. Such a lower bound was improved to be 2.218^n by Tyrrell (Tyrrell 2022). However, Romera-Paredes et al. (Romera-Paredes et al. 2024) managed to improve the lower bound by using an LLM with an evaluator, to 2.2202^n.

Romera-Paredes et al. employ a procedure called *FunSearch*, short for "searching in the function space," which pairs a pretrained large language model with a systematic evaluator (ibid, p. 468). *FunSearch* discovered a larger cap set in dimension (n = 8) than previously known: formerly, it was known that the best known cap set for n = 8 is of size 496; the authors consequently discovered a cap set of size 512 in n = 8. Moreover, it is important to emphasize that FunSearch does not search by directly enumerating candidate sets. Instead, it iteratively recombines "several programs sampled from the programs database"; this composite prompt is fed to the pretrained LLM to generate new programs, which are then evaluated, scored, and, if correct, added back into the database, thereby "closing the loop." (ibid., p. 469) In that way, not only the set of 512 eight-dimensional vectors was discovered, but also (and before that) "a program that generates it […]. [Only by] inspecting the code, we [the authors] obtain a degree of understanding of what this set is: specifically, manual simplification of [the code] provides the construction" of this set of 512 element, which "we were able to manually construct thanks to having discovered the cap set by searching in function space" (ibid, p. 471).

In contrast to the previous example, the mathematician's or programmer's contribution here is concentrated in shaping the interaction with the system, especially through what Miao and Wang (2024, p. 27) call "prompt engineering," since "designing effective prompts is crucial for obtaining the desired results from LLMs." Romera-Paredes et al. likewise make clear that their role did not end with crafting prompts or running the search. Their understanding of the result emerged only *after* FunSearch produced a successful program: by inspecting and simplifying the discovered code, they were able to extract a humanly intelligible construction and then manually reconstruct the 512-element cap set (Romera-Paredes et al. 2024, p. 471). This case therefore exemplifies a distinctly hybrid temporal pattern: the system generates an object and a generating procedure first, and mathematical intuition enters in a retroactive mode, as interpretation and reconstruction of what the system has found.

FunSearch, thus, presents a stronger form of human-machine hybridity than the cases of knot theory or euclidean geometry. In the FunSearch case, human agents prompt the LLM which in turn generates code that may surprise the researchers, leading them to reconceptualize the problem and manually reconstruct the discovered set. Intuition here is neither fully anticipatory nor purely retroactive, but mutually reshaped: human prompting guides machine exploration, while machine outputs which are not a specific solution but rather a program, reshape human understanding. This can be viewed as an "extended" cognitive loop (Longo & Viarouge 2010; Clark & Chalmers 1998; Rupert 2009) or distributed cognition (Hutchins 2000; Osbeck & Nersessian 2013), describing a co-constitution in which human and non-human agency are reciprocally formative.

4.4 Critical Co-constitution: The case of the bunkbed conjecture

Not every attempt to use neural networks or LLMs leads necessarily to a breakthrough[26]. The usage of LLMs may very well lead to wrong conclusions concerning the discussed problem, even if it is believed that the solution to such a problem or conjecture is intuitive. The fourth example turns to the bunkbed conjecture, which demonstrated such a case: this conjecture deals with two graphs, initially identical, imagined as posited one above the other, when all of their vertices are connected (hence obtaining a structure similar to a bunkbed), though afterwards some of the edges in the corresponding graphs are deleted according to a certain probabilistic method. Picking two vertices in the bottom graph, one calculates the probability that there will be a path between the two. Looking again at the same two vertices, one proceeds as follows: for one of them, we choose the vertex directly above it at the top graph. One now asks whether there is a path from the starting vertex at the bottom graph to the ending vertex at the top graph. The conjecture says that the probability of finding the path on the bottom graph is always greater than or equal to the probability of finding the path that jumps to the top graph.

The conjecture was introduced by Pieter Kasteleyn in 1985[27]. As Nikita Gladkov, Igor Pak, and Aleksandr Zimin remark, it is "both natural and intuitively obvious," yet it "has defied repeated proof attempts" and is known only in a few special cases (Gladkov, Pak & Zimin 2024, p. 1). In their effort to disprove it, they experimented with machine-learning methods, training a neural network to generate examples that appeared to support the conjecture's validity. They ultimately concluded, however, that the Bunkbed Conjecture has "unique features" that make it "very poorly suited for computer testing" (ibid, p. 10). The conjecture was instead overturned by what they describe as a "formal argument."

What matters for our purposes is the configuration of intuition in this case. Unlike the previous examples[28], where intuition is either partially outsourced to the system or re-enters retroactively as an explanation of what the system produced, Gladkov, Pak, and Zimin treat both the original, long-standing intuition in favor of the conjecture and the apparent "support" generated by machine learning as epistemically secondary. Here the mathematician's role is not to refine, interpret, or

[26] This was seen already with AlphaGeometry's inability to solve some problems, see footnote 21.

[27] See: (van den Berg and Kahn 2001, p. 124). The authors, van den Berg and Kahn, note that this conjecture was "a conjecture we learned from the late P. W. Kasteleyn" (personal communication, circa 1985).

[28] Note that this example is not discussed by Miao and Wang (2024).

curate machine outputs, but to resist being guided by them and to reassert a deductive route that ultimately defeats an intuition that had looked compelling for decades.

The bunkbed case points to a different mode of human–machine interaction, one we can describe as critical co-constitution. Rather than serving as a guide to discovery or as a resource for retroactive understanding, the machine-generated outputs in this episode functioned as a pressure in the opposite direction, amplifying a line of thought that ultimately had to be rejected. In this respect, the case casts intuition not as a dependable orienting capacity but as something that can mislead, recalling earlier moments in the history of mathematics in which compelling "intuitive" methods were later criticized as epistemically unstable, such as Weil's and Zariski's reactions to the methods employed by the Italian school of algebraic geometry. The broader point is that co-constitution is not always harmonious; sometimes the human role is precisely to withstand machine influence.

5. Relocating Intuition

The four cases examined above suggest that we are entering a new phase in the history of mathematical practice in which AI-based technologies demand a revision of how mathematical intuition is understood and situated. The central change is not that human intuition disappears, nor that machines "replace" a human faculty. Rather, intuition is being relocated within the temporal and agential structure of mathematical work. Historically, intuition has often been cast as what comes first: the anticipatory capacity that orients inquiry, suggests promising moves, and makes formalization possible. In AI-based settings, by contrast, results and constructions can be generated without prior human anticipation, and intuition is therefore often required to operate after the fact as interpretation, assimilation, critique, and reconstruction. This is consistent with the historical plasticity traced in Section 3, but it also introduces a distinctive contemporary twist: intuition is no longer only what initiates discovery; it is also what makes machine-produced outcomes intelligible and epistemically usable.

This relocation is not a rupture but a redefinition of roles within an expanded epistemic framework of mathematical division of labor. Across the four cases, the human mathematician remains central, but not primarily as the sole originator of the mathematical path from conjecture to proof. In the AlphaGeometry case, the system can produce auxiliary constructions that are later experienced as "non-obvious," leaving the mathematician with the task of retroactively supplying the intelligibility that traditionally accompanies geometric insight. In the knot theory case, the system's role is to detect correlations and propose patterns across massive datasets, while the mathematician's anticipatory intuition continues to curate what counts as mathematically significant, what is worth conjecturing, and what deserves proof. In the FunSearch cap-set case, the system does not simply output a candidate object; it outputs a generating program, and understanding emerges through a human act of inspecting, simplifying, and reconstruing the code into a mathematical construction. Finally, the bunkbed conjecture shows that hybridization can be critical rather than harmonious: machine outputs can amplify an intuitively compelling direction that must be resisted, and the mathematician's role becomes one of maintaining deductive discipline against misleading pressures. Taken together, these examples show that the mathematician's centrality is preserved, but it is increasingly expressed as co-interpretation, meaning-curation, and problem-space design rather than as solitary discovery.

This shift has consequences for how we should define mathematical intuition today. Section 3 showed that contemporary literature already treats mathematical intuition as an umbrella term for multiple practice-sensitive capacities rather than a single psychological faculty. Our claim is that AI-based mathematical practice requires a more sophisticated version of that umbrella. A definition adequate to current conditions should not only enumerate multiple cognitive roles, but also build in the temporal and agential reconfiguration introduced by AI. In light of the four cases, we propose the following working definition: mathematical intuition is a family of practice-embedded capacities for orienting, evaluating, and rendering intelligible mathematical moves and outcomes, where these capacities may operate both anticipatorily (guiding what to try, pursue, or prove) and retroactively (interpreting, reconstructing, and critiquing results produced within hybrid human–machine workflows). On this definition, intuition is not an inner flash that belongs exclusively to an individual human mind; it is a set of epistemic practices distributed across people, tools, representations, and evaluative procedures, and it may be triggered by outputs that originate outside the human agent's anticipatory horizon.

The philosophical upshot is that intuition should no longer be treated as a stable marker separating human mathematical cognition from machine calculation. Instead, intuition becomes one of the key sites at which collaboration reorganizes mathematical agency and understanding. In AlphaGeometry, intuition is relocated toward reconstruction and explanation. In knot theory, it remains anticipatory but is exercised through selection and sense-making over machine-proposed patterns. In FunSearch, intuition is reshaped inside an extended loop in which machine-generated code alters what humans can recognize as a construction. In the bunkbed episode, intuition includes the capacity to resist and discipline what looks compelling, even when computational evidence appears to reinforce it. These modes do not eliminate the classical picture of intuition, but they pluralize and redistribute it. What emerges is a conception of mathematical intuition as a hybrid, sometimes retroactive, practice of understanding and interpretation across human and machine actors, one that is increasingly central precisely because AI expands the space of what can be generated beyond what humans can readily anticipate.

Clearly, intuition is not the only concept pressured by AI-based mathematics. Agency and understanding, among others, are tightly entangled with it, because shifts in when and how intuition operates also shift who is doing what in mathematical work, and what it means to grasp a result. We will not offer a full analysis of these concepts here, but two brief remarks indicate how the relocation of intuition bears on both.

First, the relocation of intuition re-specifies mathematical agency. Agency has traditionally been attributed primarily to the human mathematician, with machines treated as passive instruments (Mancosu et al. 2005; Ferreirós 2016; Hamami 2023). In that framework, intuition marks agency by guiding discovery, selecting promising paths, and recognizing significance (de Toffoli & Giardino 2014; Larvor 2019). AI-based tools complicate this alignment by producing constructions and correlations that are not straightforwardly reducible to a mathematician's explicit intentions. As a result, human agency is preserved but shifts toward governance of a hybrid pipeline: deciding what to pursue, what to accept, and how to frame machine outputs as mathematically meaningful or misleading.

Second, the same relocation bears on understanding. Philosophers have long distinguished understanding from merely having a correct proof, emphasizing intelligibility and "seeing why"

(Riggs 2003; Grimm 2006; Avigad 2008; De Regt 2017). AI-assisted mathematics often produces correct outputs whose rationale is not immediately transparent, so mathematicians must work backward to supply explanation, salience, and conceptual integration. In this sense, intuition becomes a central vehicle of understanding precisely by operating retroactively: reconstructing motivations for constructions, discerning which mathematical correlations are explanatory, translating machine code into a communicable construction, and recognizing the limits of machine-guided plausibility.

These changes, together with the cases discussed above, indicate that mathematical intuition is being reconfigured alongside other concepts, taking on roles it was not previously expected to occupy. Its distinctive contribution increasingly lies in the work of *integration*: converting outputs into reasons, patterns into claims, and procedures into communicable constructions, while maintaining the critical standards that determine what becomes part of mathematics. In this sense, AI does not remove intuition (nor agency, or understanding, or other related concepts) from the practice; it makes visible a broader profile of intuition as an epistemic activity that spans the full arc of inquiry, including forms that are activated only once a result is already on the table.

6. Concluding Remarks

Mathematics has never been an entirely "mental" pursuit. From diagrams in Euclid to chalkboards in the 20th century, mathematical reasoning has always been mediated by materials, representations, and tools (Barany & MacKenzie 2014; Manders 2008). Notwithstanding this mediation, it has been assumed that the creation of (mathematical) symbols is inherently human (i.e., mathematical texts originate solely from human thought) and that the material tools for writing and computation were merely passive instruments (see: Bajohr & Hiller 2024). What distinguishes the present moment is that these tools no longer merely inscribe or extend human thought, but actively generate constructions, conjectures, and proofs that reconfigure what counts as intuitive. If earlier 'crises' of intuition emerged around non-Euclidean spaces, 'monstrous' functions, or computer-assisted proofs, the novelty today is that outputs, some of them – but not necessarily all – are considered non-intuitive, are now distributed between human and non-human agents. Large language models (LLMs) challenge these assumptions of the tools' passivity, not only by contributing directly to the creation of mathematical works but also by undercutting the idealistic and romantic notion of mathematical authorship as an expression of human intuition. They undermine the belief that mathematical practices rely on a non-formalized and non-formalizable intuition, which can only later be translated into words and symbols, further reshaping the notion of authorship in mathematics. This requires not the abandonment of intuition, but its repositioning into a hybrid space where humans and machines jointly constitute mathematical knowledge.

This relocation highlights a deeper point: intuition has always been plastic, but its current reconfiguration is qualitatively novel. It no longer belongs exclusively to the mathematician's "inner eye," nor does it dissolve into brute computation. Instead, it takes the form of hybrid practices: interpretive, heuristic, reconstructive or even being 'guided' by AI-based technologies. Combining the historical perspective with current case studies showed us that such views are not entirely new, but it is clear that a new partnership in the form of hybrid collaborations emerges, and hence the role of the mathematician and her intuition changes. The mathematician remains central, but as a co-constitutor of knowledge within systems that are partly non-human. Intuition persists, but as a relational property of human–machine constellations rather than a solitary, solely-human faculty. The question obviously still remains whether such AI-based processes can be also considered as 'genuine mathematics'. While we tend to reply positively to this question, as they can solve complex problems by formalizing mathematical language into systems and systematically generating and verifying proofs (Vega 2023; Song, Yang & Anandkumar 2024), one issue should be emphasized: despite all the advancement presented with AI-based technologies, and taking also its limitations presented above into account, such technologies cannot (yet) create new mathematical concepts. They indeed present new, unexpected constructions and proofs, but they are not (yet) in a position to completely change or restructure conceptually mathematical domains.

Recognizing this shift matters not only for mathematics but for philosophy and science more broadly. It calls on us to refine our accounts of agency, understanding, and creativity in light of distributed epistemic systems (Pickering 2010; Medina & Harding 2025). If mathematics is a privileged site for witnessing the transformation of intuition, then it is also a harbinger for how AI will reshape knowledge in other domains. This article hence aimed to theorize intuition not as what AI lacks, but its different epistemological status when humans and machines collaborate in the production of intelligibility.

References:

Appel, K.I. and Haken, W., 1989. Every planar map is four colorable (Vol. 98). American Mathematical Soc.

Avigad, J. (2024a). Mathematics and the formal turn. *Bulletin of the American Mathematical Society*, 61(2), 225-240.

Avigad, J. (2024b). Automated Reasoning for Mathematics. In: Benzmüller, C., Heule, M.J., Schmidt, R.A. (eds) Automated Reasoning. IJCAR 2024. *Lecture Notes in Computer Science,* vol 14739. Springer, Cham.

Avigad, J. 2008. Understanding proofs. https://kilthub.cmu.edu/articles/Understanding_Proofs/6493034/files/11937752.pdf (last accessed on: 15 September 2025)

Barany, M.J. & MacKenzie, D. (2014). Chalk: Materials and concepts in mathematics research. *Representation in Scientific Practice Revisited*, 107-130.

Bajohr, H. & Hiller, M. (2024). Das Subjekt des Schreibens. Einleitung. *Text + Kritik* X/24, 5-16.

Berg, Á. & Tanswell, F. (2025). The Philosophical Prospects of Large Language Models in the Future of Mathematics. forthcoming in the Annals of Mathematics and Philosophy.

Borel, É. (1922). *Méthodes et problèmes de théorie des fonctions*. Gauthier-Villars.

Bourbaki, N. 1949. "Foundations of Mathematics for the Working Mathematician." The Journal of Symbolic Logic 14 (1): 1–8. https://doi.org/10.2307/2268971

Bussotti, P. & Pisano, R. (2015). The Geometrical Foundation of Federigo Enriques' Gnoseology and Epistemology, *Advances in Historical Studies* 4, 118–145.

Clark, A. & Chalmers, D. (1998). The extended mind. analysis, 58(1), 7-19.

Cooney, J.B. (1991). Reflections on the origin of mathematical intuition and some implications for instruction. Learning and Individual Differences 3(1), 83-107.

Corry, Leo. 2004. *Modern Algebra and the Rise of Mathematical Structures*. Basel: Springer.

Chudnoff, E. (2020). Forming Impressions: Expertise in Perception and Intuition. Oxford: Oxford University Press.

Chudnoff, E. (2019). In search of intuition. Australasian Journal of Philosophy, 98, 465–480.

Chudnoff, E. (2014). Intuition in Mathematics. In Linda Osbeck & Barbara Held (eds.), *Rational Intuition*. Cambridge University Press.

Chudnoff, E. (2011). The Nature of Intuitive Justification. *Philosophical Studies* 153 (2):313 - 333.

D'Alessandro, W. and Stevens, I., 2024. Mature intuition and mathematical understanding. The Journal of Mathematical Behavior, 76, p.101203.

Davies A, Veličković P, Buesing L, Blackwell S, Zheng D, Tomašev N, Tanburn R, Battaglia P, Blundell C, Juhász A, Lackenby M, Williamson G, Hassabis D, Kohli P. (2021). Advancing mathematics by guiding human intuition with AI. *Nature* 600(7887), 70–74.

Davis E. (2021) Deep learning and mathematical intuition: a review of (Davies et al. 2021). https://arxiv.org/abs/2112.04324

Davis E. (2023) Mathematics, word problems, common sense, and artificial intelligence. https://arxiv.org/abs/2301.09723

Dehaene, S., 2009. Origins of mathematical intuitions: The case of arithmetic. Annals of the New York Academy of Sciences, 1156(1), pp.232-259.

De Toffoli S & Giardino V (2014) Forms and roles of diagrams in knot theory. Erkenntnis 79(3): 829–842.

De Toffoli, S., (2021). Reconciling rigor and intuition. Erkenntnis, 86(6), 1783-1802.

De Toffoli, S. & Fontanari, C. (2022). Objectivity and rigor in classical Italian algebraic geometry. Noesis, (38), 195-212.

Detlefsen, M. (1993). Hilbert's Formalism. *Revue Internationale de Philosophie* 47(186), 285–304.

Detlefsen, M. (2005). 'Formalism', in Stewart Shapiro (ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford: Oxford University Press, 236–317.

De Regt, H. W. (2017). *Understanding scientific understanding*. Oxford university press.

Enriques, F. (1938). *Le matematiche nella storia e nella cultura*. Bologna: Zanichelli.

Epple, M. (2013). Die Entstehung der Knotentheorie: Kontexte und Konstruktionen einer modernen mathematischen Theorie. Springer-Verlag.

Epple, M. (2022). 'Analogies,' 'Interpretations,' 'Images,' 'Systems,' and 'Models': Some Remarks on the History of Abstract Representation in the Sciences Since the Nineteenth Century. In: Friedman, M., Krauthausen, K. (eds) Model and Mathematics: From the 19th to the 21st Century. Trends in the History of Science. Birkhäuser, Cham. https://doi.org/10.1007/978-3-030-97833-4_9

Fawzi, A., Balog, M., Huang, A., Hubert, T., Romera-Paredes, B., Barekatain, M., Novikov, A., R. Ruiz, F.J., Schrittwieser, J., Swirszcz, G. & Silver, D., (2022). Discovering faster matrix multiplication algorithms with reinforcement learning. Nature, 610(7930), 47-53.

Ferreirós J. (2016). *Mathematical knowledge and the interplay of practices*. Princeton University Press, Princeton.

Frege, G. (1903), *Grundgesetze der Arithmetik, Begriffsschriftlich Abgeleitet* (Volume II), Jena: Pohle; reprinted 1962, Hildesheim: George Olms.

Gastaldi, J.L. (2024). How to Do Maths with Words: Neural Machine Learning Applications to Mathematics and Their Philosophical Significance. In: Sriraman, B. (eds) Handbook of the History and Philosophy of Mathematical Practice. Springer, Cham. https://doi.org/10.1007/978-3-031-40846-5_142

Giardino, Valeria (2018). Manipulative imagination: how to move things around in mathematics. Theoria : An International Journal for Theory, History and Fundations of Science 33 (2):345-360.

Gladkov, N., Pak, I. & Zimin, A. (2024). The bunkbed conjecture is false. https://doi.org/10.48550/arXiv.2410.02545

Grimm, S. (2006). Is Understanding a Species of Knowledge? *British Journal for the Philosophy of Science* 57(3), 515–535.

Hahn, H., (1980 [1933]). The crisis in intuition. In Empiricism, logic and mathematics: Philosophical papers, 73-102. Dordrecht: Springer Netherlands.

Hamami, Y. (2023). Agency in Mathematical Practice. In: Sriraman, B. (eds) Handbook of the History and Philosophy of Mathematical Practice. Springer, Cham. https://doi.org/10.1007/978-3-030-19071-2_48-1

Hersh, Reuben (2011) "Mathematical Intuition (Poincaré, Polya, Dewey)," The Mathematics Enthusiast: Vol. 8 : No. 1 , Article 3.

Hiller, M. (2023) Es gibt keine Sprachmodelle, in: Davide Giuriato, Claas Morgenroth und Sandro Zanetti (ed.), Noten zum »Schreiben«. Für Martin Stingelin zum 60. Geburtstag, Paderborn, 279–285.

Hutchins, E., (2000). Distributed cognition. International encyclopedia of the social and behavioral sciences, 138(1), pp.1-10.

Janiak, A., (2022). Kant's Views on Space and Time, The Stanford Encyclopedia of Philosophy (Summer 2022 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2022/entries/kant-spacetime/>.

Kantamneni, S. & Tegmark, M. (2025). Language models use trigonometry to do addition. https://doi.org/10.48550/arXiv.2502.00873

Klein, F. (1911 [1893]). The Evanston Colloquium. Lectures on mathematics. American Mathematical Society, New York.

Lajos, J. (2023). An updated conceptualization of the intuition construct for mathematics education research. Journal of Mathematical Behavior, 71, 1–19.

Larvor B. (2019) From Euclidean geometry to knots and nets. Synthese 196(7): 2715–2736

Lehmer, D.H., Lehmer, E. & Vandiver, H.S., (1954). An application of high-speed computing to Fermat's last theorem. Proceedings of the National Academy of Sciences, 40(1), 25-33.

Leibniz, G. W. (1923–). Sämtliche Schriften und Briefe. Berlin-Brandenburgische Akademie der Wissenschaften, Akademie der Wissenschaften zu Göttingen.

Longo, G. and Viarouge, A., (2010). Mathematical intuition and the cognitive roots of mathematical concepts. Topoi, 29(1), 15-27.

Mac Lane, Saunders. 1971. *Categories for the Working Mathematician*. New York: Springer.

MacKenzie, D., (2004). *Mechanizing proof: computing, risk, and trust*. MIT Press.

Mancosu P., Jørgensen K. F., & Pedersen S. A. (eds) (2005) Visualization, explanation and reasoning styles in mathematics. Dordrecht: Springer.

Manders, K. (2008). The Euclidean Diagram (1995), in Paolo Mancosu (ed.), The Philosophy of Mathematical Practice. Oxford: Oxford University Press , 80-133.

Medina, L. R. & Harding, S. (eds.) (2025). Decentralizing Knowledges: Essays on Distributed Agency. Duke University Press.

Mehrtens, H. (1990). *Moderne Sprache, Mathematik: Eine Geschichte Des Streits Um Die Grundlagen Der Disziplin Und Des Subjekts Formaler Systeme*. Frankfurt am Main: Suhrkamp.

Miao, Q., Wang, FY. (2024). Artificial Intelligence for Science (AI4S). SpringerBriefs in Service Science. Springer, Cham.

Osbeck, L. M., & Nersessian, N. J. (2013). Situating distributed cognition. Philosophical Psychology, 27(1), 82–97. https://doi.org/10.1080/09515089.2013.829384

Pantsar, Markus. (2024). Theorem proving in artificial neural networks: new frontiers in mathematical AI. Euro Jnl Phil Sci 14, 4 https://doi.org/10.1007/s13194-024-00569-6

Parshina, K. (2023). "Philosophical Assumptions Behind the Rejection of Computer-Based Proofs" KRITERION – Journal of Philosophy 37, no. 2-4, 105-122.

Pedwell, C. (2024) Speculative machines and us: more-than human intuition and the algorithmic condition, Cultural Studies, 38:2, 188-218.

Pellegrino, G. (1970). Sul massimo ordine delle calotte in $S_{4,3}$, Matematiche (Catania), 25: 149-157

Pickering, A. (2010). *The mangle of practice: Time, agency, and science*. University of Chicago Press.

Poincaré, H., (1899). Des fondements de la géométrie: à propos d'un livre de M. Russell. Revue de métaphysique et de morale, 7(3), 251-279.

Poincaré, H., (1905). Les mathématiques et la logique. Revue de métaphysique et de morale, 13(6), 815-835.

Poincaré, H. (1914 [1908]). *Science and Method*. Trans. Maitland, F.. London et. al: Thomas Nelson and Sons.

Poincaré, H. (1913). *The Foundations of Science*. New York: The Science Press.

Putnam, H., & Benacerraf, P. eds. (1984). Philosophy of Mathematics: Selected Readings. Second edition. Cambridge: Cambridge University Press.

Riggs, Wayne D. (2003). Understanding 'Virtue' and the Virtue of Understanding. In Michael Raymond DePaul & Linda Trinkaus Zagzebski, *Intellectual virtue: perspectives from ethics and epistemology*. New York: Oxford University Press. pp. 203-226.

Romera-Paredes, B., Barekatain, M., Novikov, A. et al. (2024). Mathematical discoveries from program search with large language models. Nature 625, 468–475. https://doi.org/10.1038/s41586-023-06924-6

Rupert, R.D., (2009). Cognitive systems and the extended mind. Oxford University Press.

Samuel, N., (2020). Die Form des Chaos: Bild und Erkenntnis in der komplexen Dynamik und der fraktalen Geometrie. Brill Fink.

Song, P., Yang, K., & Anandkumar, A. (2024). Towards Large Language Models as Copilots for Theorem Proving in Lean. ArXiv, abs/2404.12534.

Testolin, A. (2024). Can Neural Networks Do Arithmetic? A Survey on the Elementary Numerical Skills of State-of-the-Art Deep Learning Models. *Applied Sciences* 14, no. 2: 744. https://doi.org/10.3390/app14020744

Trinh, Trieu H., Wu, Yuhuai, Le, Quoc V. et al. (2024) "Solving olympiad geometry without human demonstrations". Nature 625, 476–482. https://doi.org/10.1038/s41586-023-06747-5

Turing, A. (2004 [1948]). Intelligent machinery (1948), In *The essential Turing*. 395–432. Oxford: Oxford University Press.

Tyrrell, F., (2022). New lower bounds for cap sets. arXiv preprint: arXiv:2209.10045.

van den Berg, J. & Kahn, J. (2001). A correlation inequality for connection events in percolation. Annals of Probability. 29 (1): 123–126.

Van-Quynh, A., (2017). Intuition in mathematics: A perceptive experience. Journal of Phenomenological Psychology, 48(1), 1-38.

Van-Quynh, A., (2019). The three formal phenomenological structures: A means to assess the essence of mathematical intuition. Journal of Consciousness Studies, 26(5-6), 219-241.

Vega, F. (2023) Lean on Goldbach's conjecture. ⟨hal-04194039⟩ https://hal.science/hal-04194039/

Volkert, K. (1986). *Die Krise der Anschauung: eine Studie zu formalen und heuristischen Verfahren in der Mathematik seit 1850*, Göttingen: Vandenhoeck & Ruprecht.

Volkert, K. (1987) Die Geschichte der pathologischen Funktionen – Ein Beitrag zur Entstehung der mathematischen Methodologie. *Archive for History of Exact Sciences*, 37/3, 193–232.

Whitehead, A. N. & Russell, B. (1910), Principia Mathematica Cambridge: Cambridge University Press.

Zermelo, E. (1908). Investigations into the foundations of set theory. I. Mathematische Annalen , 65 (2), 261-281.