# Chapter 1
# Introduction: From Dissent to Dismissal: Brouwer's Journey in Mathematical Annals

**Abstract** Scientific revolutions are often told as stories of triumph, yet histories of failure to revolutionize science are equally revealing. This introduction begins with L.E.J. Brouwer's attempt to re-found mathematics on intuitionistic grounds, a bold program that mobilized worldwide debates in the 1910s and 1920s but ultimately lost ground. By foregrounding this "unsuccessful" revolution, the book highlights how resistance, controversy, and eventual dismissal illuminate the social fabric of science. In dialogue with post-Kuhnian models of scientific change, Brouwer's story illustrates how intellectual projects are shaped not only by conceptual rigor but by institutional politics, group loyalties, and interpersonal tensions. The chapter frames intuitionism as more than a failed school: it is a case study in the dynamics of dissent, showing how the mathematics of the early twentieth century was forged at the crossroads of ideas and communities.

At the turn of the twentieth century, a whirlwind of transformation swept the world, leaving an indelible mark on history. Wars reshaped the world order, social movements fought for equality, and technological leaps altered the fabric of society. Aviation pioneers took flight, while groundbreaking scientific theories challenged established norms. For example, the field of psychology was transformed by the emergence of Freud's psychoanalysis and Watson's behaviorism, reshaping the understanding and treatment of human behavior. Physics was revolutionized by Albert Einstein's theory of relativity, which fundamentally altered our understanding of time, space, and gravity, moving away from the Newtonian mechanics that had dominated for centuries. Meanwhile, in biology, the rediscovery of Gregor Mendel's work on genetics at the beginning of the century ignited the field of genetics, shifting the study of heredity to a molecular level and setting the stage for modern biology. Each of these examples is a successful "breakthrough"—a discovery that solves a problem or overcomes a barrier and eventually transforms science.

However, while successful transformations often steal the spotlight, I embark on a journey in this book to illuminate a lesser-known tale—a failed revolutionary attempt in the realm of mathematics, where one mathematician, named Brouwer, aimed to solve a fundamental problem in the foundations of the discipline, sparking



K. Kish Bar-On, *Intuitionism Resocialized*, Studies in History and Philosophy of Science 66, https://doi.org/10.1007/978-3-032-05839-3_1

a decade-long debate. This story, which didn't seize victory but never faded from discussion, serves as a unique lens through which we can reflect on the nuances of historical narratives and the impact of unsuccessful revolutions on our collective understanding.

Success in science is often seen by the scientific community as an advancement that resolves a challenge, shifts existing frameworks, and significantly alters our understanding. Failure, on the other hand, doesn't necessarily mean a lack of contribution. Consider Lamarck's early evolutionary theory, which incorrectly suggested that traits acquired during an organism's life could be passed on to offspring. Though ultimately replaced by Darwinian evolution, Lamarck's ideas spurred debate, influencing the development of evolutionary thought and highlighting how even unsuccessful attempts can push the boundaries of knowledge.

A relevant aspect of successes and failures in science is that they are not simply binary; they often interact in ways that drive knowledge forward. Successful theories can emerge from the ashes of failed ideas, and failed attempts can illuminate new paths for exploration. Lamarck's theory, though flawed, sparked critical debates that laid groundwork for future evolutionary theories. Thus, success and failure in science can complement and influence each other, showing that scientific progress is a continuous dialogue, one marked by ongoing discussions, where even unsuccessful efforts contribute to the broader understanding of the natural world.

Unsuccessful scientific revolutions, though often relegated to the margins of historical narratives, are vital to our comprehensive understanding of scientific evolution. They underscore the paramount importance of consensus and the rigorous process of peer review, illustrating that science progresses through a collaborative rather than solitary endeavor. These attempts frequently clash with entrenched paradigms, revealing the conservative nature of scientific communities that favor stability and stepwise innovation over radical departures. Beyond the realm of empirical data and logical constructs, the fate of these revolutions is also deeply intertwined with sociocultural currents that can either hinder or hasten scientific acceptance. The tales of these not-quite-successful ventures serve as a beacon for future scientists, embodying the courage required to question the status quo and the resilience to withstand criticism. Taken together, these elements weave a complex tapestry that portrays scientific advancement not merely as a sequence of triumphant discoveries but as a nuanced, deeply social endeavor, marked by debate, collaboration, and disagreement at times.

Some see mathematics as unique among the sciences for its potential to be revolutionized, though such revolutions are often considered less frequent compared to other fields (Ferreira and Silva 2020; Wigderson 2019). It is largely assumed that mathematics, being cumulative and axiomatic, tends to refine or build upon existing theories rather than completely overturning them (Crowe 1975). Consider calculus for example: it emerged from centuries of earlier work, including methods by Greek mathematicians like Archimedes for approximating areas and medieval studies of sequences and infinitesimals. These concepts were extended into a formal system by Newton and Leibniz, creating powerful tools for mathematics and physics. Calculus is grounded in the axioms of algebra and geometry, which provided a logical framework for its principles, such as limits, derivatives, and integrals. Rather than discarding the foundational theories of algebra and geometry, calculus built on them, offering new ways to understand motion, curves, and areas. Later refinements by mathematicians like

Cauchy and Weierstrass further formalized calculus, enhancing its rigor without disrupting its core principles, exemplifying the continuous and cumulative nature of mathematical developments. Such gradual developments are thought to provide a sense of stability, where changes are viewed more as expansions than disruptions. Unlike empirical sciences, where new experimental data can drastically alter existing paradigms, mathematics deals with abstract concepts that are generally seen as universally true and not dependent on physical evidence. As a result, mathematical truths are often perceived as enduring indefinitely, unless proven otherwise.

Progress in mathematics is often characterized by deeper insights into known theories, the discovery of connections between various domains, or the development of more comprehensive models that unify disparate areas (Thurston 1995; Weisgerber 2023). While these advances are revolutionary in scope, they may not be perceived as such because they build upon, rather than contradict, existing knowledge. Additionally, the stability of mathematical theories and the meticulous process required to verify new propositions contribute to the appearance of a slower pace of revolutionary change. The rigorous scrutiny needed for consensus and acceptance means that new ideas take time to be fully recognized and assimilated, adding to the impression that mathematics evolves more through steady progression than through abrupt upheavals.

Successful or not, every revolution attempt has historical roots. The history of mathematics is full of false starts and undertakings that came to nothing: disproved hypotheses, inadequate techniques, misguided theorizing, and much more. All of these belong in what Thomas Kuhn famously dubbed "normal science" (Kuhn 1962). Kuhn drew our attention to the constitutive role played by a science's shared normative framework in governing the day-by-day work of scientists and its scientific assessment. However, his famous book failed to account for the rationality of framework replacement. He spoke there of the pressure exerted on a framework by accumulated anomalies and their attraction to the ambitious upstarts of the field. However, in his well-known likening of paradigm shifts to gestalt switches and religious conversions, he failed to account for the interpersonal aspect of such transitions: how interactions between individuals, communities, groups, and norms affect the trajectory of a revolutionary new theory.

Kuhn's two-tier model of scientific knowledge and scientific work, and especially his focus on scientific revolutions, have given rise to a broad array of responses—some dismissive of his approach, some more constructively critical. In this book, I closely examine four post-Kuhnian accounts of scientific sea changes of the latter category, each shares at least something of Kuhn's neo-Kantianism. Two of them were designed specifically to account for framework transitions in mathematics, one to account for the history of time-space theories in physics, and one, though more general than the other three, was applied by its author to a detailed study in the history of algebra.

Yet unlike other works in the field, the idea here is to put the four accounts to the test of an *unsuccessful* twentieth century attempt to revolutionize mathematics. I am referring to the school of intuitionism, that was established by the Dutch mathematician Luitzen Egbertus Jan Brouwer, and played a major role in the debate about the foundations of mathematics from its inception.

Brouwer's intuitionism is a philosophical-mathematical approach that expands on the idea that mathematics is a human product. It characterizes mathematics as a collection of internally coherent mental constructs, offering an alternative to classical mathematics. In contrast to the Platonistic view that regards mathematical entities as independent Platonic objects, or the formalist-finitist approach that viewed mathematical entities as elements that can be reduced to formulas and axioms, intuitionism conceives them as mental constructions created in the mind. Given that these mental constructions are shaped by human creators situated in specific temporal, spatial, cultural, and societal contexts, intuitionism becomes a particularly intriguing case study for exploring the impact of group dynamics on individual mental constructions and understanding the interconnection between communal knowledge and intrasubjective knowledge.

The aim of this book is twofold. The first is to better understand by means of the four accounts why Brouwer's intuitionistic endeavor was eventually abandoned by the mathematical community. The second aim is to shine a light on specific mathematicians and how they responded to Brouwer's ideas. Through these aims, I uncover insights into how the interplay of individuals, groups, norms, and institutions shape mathematical knowledge. Each of the four models address these goals somewhat differently, and together they combine to provide a more complex understanding of such moments than any of the four alone, can offer. This book thus presents, for the first time, a full-fledged socially-oriented history of the rise and fall of Brouwer's intuitionism. In doing so, it takes an initial stride toward a comprehensive, socially oriented narrative of the history of mathematics, encapsulating the intricate connections between individual mathematicians and the communities they inhabit.

## 1.1 The Emergence and Demise of Brouwer's Intuitionism

First, a word of clarification is in order. This book is not about Brouwer the person. There are several insightful and comprehensive accounts of Brouwer's life, personality, and work; most notable is Dirk van Dalen's *L.E.J Brouwer—Topologist, Intuitionist, Philosopher* (van Dalen 2013). I have practically nothing to add to these compelling works. The focus of the current book is on Brouwer's intuitionistic program, and on the reactions of the community and specific individuals, to it. Having said that, I find it impossible to consider a scientist's work as completely separate from his personal and environmental circumstances. Hence, the following paragraphs will sketch, in a very brief and concise manner, the main characteristics of the rise and fall of Brouwer's intuitionism with a glimpse of Brouwer's personal life. Nevertheless, throughout the rest of the book, the personal aspects of Brouwer's life will remain mostly in the shadows.

Luitzen Egbertus Jan Brouwer was born in 1881 at Overschie, a Rotterdam neighborhood, the second-largest city in the Netherlands. After a year, the family moved to Medemblik, where Brouwer spent his early school years, and a decade later, they moved again, this time to Haarlem, the capital of the province of North-Holland. At the age of 16, Brouwer started his academic studies at the faculty of Mathematics and Sciences at the University of Amsterdam (UVA), where he first met the mathematics professor Diederik Johannes Korteweg and started to attend his lectures. The correspondence between Brouwer and his close friend Carel S. Adama van Scheltema during Brouwer's years at UVA portrays Brouwer as a sensitive and devoted young man, often prone to mood swings and occasional depression (van Dalen 2011).

After obtaining his doctorandus (a Dutch academic title equivalent to M.A. or M.Sc. degree) in 1904, Brouwer found himself torn between philosophy and mathematics as he had to reach a decision about the future direction of his scientific career. It is evident from his 1905 booklet *Life, Art and Mysticism* that Brouwer's strong philosophical views did not dissolve during his years of academic studies (Brouwer 1905). Nevertheless, he decided to pursue a mathematical dissertation under the supervision of Korteweg. The dissertation marked the turning point in Brouwer's engagement with the wider world, in the sense that Brouwer did not give up his mystic ideas, but he realized that he had to rephrase them or put them aside if he wants to make an impact as a mathematician.

Brouwer successfully defended his dissertation in February 1907. Between 1909 and 1913, Brouwer dedicated his efforts to founding modern topology, while maintaining his intuitionistic program as a side project. He introduced significant concepts as part of classical mathematics, such as that of "mapping degree" and the so-called fixed point theorem (van Atten 2020; Brouwer 1911). After being appointed full professor ordinaries in 1913 (succeeding Korteweg) and joining the *Mathematische Annalen's* editorial board in 1914, Brouwer publicly returned to his first and foremost interest and embarked on the systematic intuitionistic reconstruction of mathematics.

During the war years, Brouwer mainly reflected on the foundations of mathematics, in particular on infinite sequences, and his deliberations during that period culminated in the publication of a series of three papers under the title "Founding mathematics independently of the logical theorem of the excluded middle" (Brouwer 1918, 1919, 1923). These papers introduced the necessary tools for a systematic practice of constructive mathematics, including the intuitionistic versions of real numbers, the continuum, and elementary topology.

In 1919, an important meeting that would eventually set in motion the foundational debate took place in Engadin. Brouwer received a visit from Hermann Weyl, David Hilbert's prominent student and philosophy enthusiast who shared Brouwer's deep concern regarding the problematic foundations of mathematics. A year later, Weyl sent Brouwer a draft of his paper, titled "On the new foundational crisis in mathematics", where he refers to Brouwer as 'the revolution' (Weyl 1921, 99).

Brouwer, who greatly admired Hilbert, was extremely flattered by Weyl's support. In September 1920, Brouwer introduced his foundational program, focusing on the core problems of mathematics, before the international mathematical forum at the *Naturforscherversammlung* at Nauheim. This "meeting of natural scientists" gathered mostly German scientists to discuss physics, mathematics, and medicine for the first time after the war. Among the 2500 participants were several prominent mathematicians, such as Felix Bernstein, Emmy Noether, Fritz Noether, Hermann Weyl, George Polya, Robert Fricke, Felix Hausdorff, Kurt Hensel, and others. Brouwer gave a talk titled "Does every real number have a decimal expansion," in which he discussed the structure of the continuum. While his Nauheim lecture remained rather neutral regarding the new intuitionistic concept of choice sequence, it did contain criticism of Hilbert's theory of the solvability of mathematical problems.[1] Such a critical presentation in an international setting set the stage for the events that followed, and the publication of Weyl's advocative paper a year later made clear that Brouwer's intuitionism could no longer be ignored. Hilbert was now forced to reply, and the foundational debate had officially started (van Atten 2017; van Dalen 1995; Hesseling 2003; Mancosu 1998; McCarty 2005; Putnam and Benacerraf 1984).

Throughout the 1920s, the foundational debate received hundreds of reactions from dozens of mathematicians (such as Alfred Errera, Paul Bernays, and Abraham Fraenkel), logicians (like Walter Dubislav and Oscar Becker), and philosophers (such as Hugo Dingler and Heinrich Scholz), among others. At the same time, Brouwer continued to develop his intuitionistic program and published another series of papers entitled "On the foundations of intuitionistic mathematics" in the *Mathematische Annalen* (Brouwer 1925, 1926, 1927). The papers presented a riper and more mature view of intuitionism, including refinements and corrections of some loose ends found in Brouwer's previous trilogy. In 1924 Brouwer published an intuitionistic proof of the fundamental theorem of algebra with his doctoral student B. de Loor (Brouwer and De Loor 1924), and in 1928 he published a refined analysis of the nature of spreads (Brouwer 1928). At the same time, while reforming mathematics and performing his academic duties, Brouwer participated in a small group of philosophers, linguistics and mathematicians named the "Significircle" whose members were Gerrit Mannoury and Frederik van Eeden, among others (Brouwer 1946; Kirkels 2013). Between 1922 and 1926, the group regularly convened to discuss spiritual and political progress through language reform (Brouwer 1937).

Brouwer's intuitionism played a major role in the foundational debate, but Brouwer's intuitionistic ideas were poorly understood within the mathematical community. Mathematicians often confused Weyl's theory with Brouwer's theory, and they generally considered Brouwer's intuitionism as "interesting news from the border provinces" (van Dalen 2013, 335) than as critical and important developments that should occupy one's mind. Meanwhile, the debate continued to spread, reaching the rest of Europe, the US, and Russia, and in 1925 a substantial contribution to intuitionistic logic was made by the Soviet mathematician Andrey Nikolaevich Kolmogorov, where he formalized parts of Brouwer's intuitionism and presented an elaborated account of the convoluted links between intuitionism, formalism, and other schools of thought (Kolmogorov 1925).

The year 1928 is often considered the turning point of the debate and the beginning of the end for Brouwer's intuitionistic program. As mentioned, Brouwer was part of the *Mathematische Annalen's* editorial board (one of its chief editors being

[1] Hilbert maintained that every mathematical problem could be solved or rejected. As he put it: "This conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no ignorabimus." (Hilbert 1902, 455).

Hilbert), for 14 years now. The *Mathematische Annalen* was the most prestigious mathematics journal at the time, and being one of its editors was considered the highest recognition for a mathematician. Despite Brouwer's diligent editorial work, in 1928, Hilbert sent a request to the other chief editors, Blumenthal, Carathéodory, and Einstein, asking their permission to remove Brouwer from the journal's editorial board due to personal conflicts and mathematical differences. After extensive correspondence involving all parties, including publisher Ferdinand Springer and his legal advisors, Hilbert's request was ultimately approved, leading to Brouwer's removal from the *Annalen's* editorial board (van Dalen 1990).

The course of events had tremendously affected Brouwer, who from that point onwards refrained from publishing in the *Mathematische Annalen* and convinced his student, Arend Heyting, to do the same (Posy 1998). As can be seen in Fig. 1.1, from 1929 onwards Brouwer never regained the energy to promote intuitionism as he did before, and his subsequent publications contributed little to the further development of intuitionism (except for his short return in 1948 to introduce his work on the creating subject).[2]

Did Brouwer intend to revolutionize mathematics? Was he a man of strategic thinking, focusing on the effective ways to promote his program and devoting all his

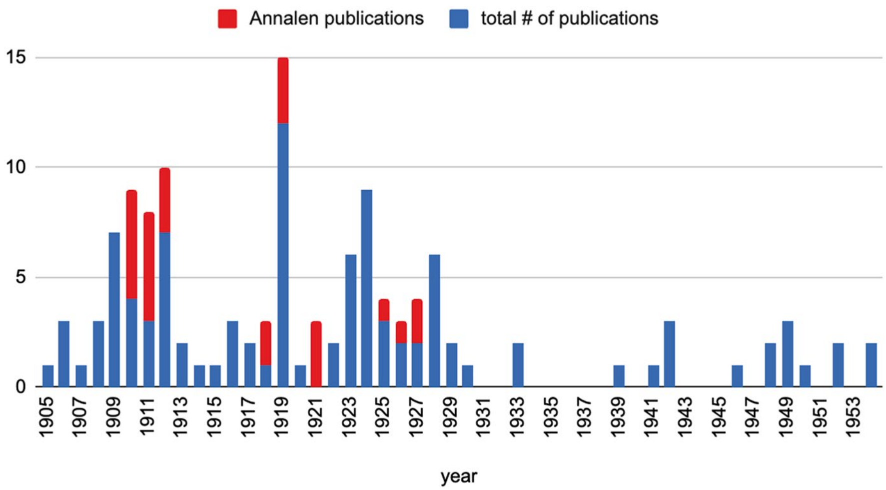


**Fig. 1.1** Brouwer's publications per year from 1905 until 1955 (Annalen publications marked in red)

[2] From a broader perspective, and compared to his successors (Heyting, Kleene, Troelstra, and their successors), Brouwer produced very little intuitionistic mathematics. His work on topology was not grounded on intuition, at least not in the sense that Brouwer understood the term in the late 1910s. Between 1918 and 1928, Brouwer wrote several important papers, introducing his intuitionistic set theory and work on choice sequences, bar theorem, and fan theorem. However, from the 1930s onwards, Brouwer ceased to publish and intentionally took a step back from the foundational debate, leaving it to be continued by the next generation.

efforts to do so? The lion's share of Brouwer scholars rightfully portray Brouwer as a mathematician rather than a politician, his interest lying solely in developing his mathematical and philosophical ideas (van Atten 2004, 2020; van Dalen 2013; Hesseling 2003; van Stigt 1990). He had very few students, out of which Arend Heyting was his "only really gifted intuitionistic student,"[3] and in the 1930s, Brouwer left him to continue the intuitionistic endeavor. Heyting formalized intuitionistic logic and mathematics, broadening intuitionism's reach but diverging from Brouwer's original vision (Heyting 1930a, b, c, 1980). Although Brouwer opposed any form of axiomatization, he supported Heyting's direction and let him lead the foundational debate (van Stigt 1990). Rather than training students to carry on his own legacy, Brouwer preferred to focus on his mathematical work, allowing Heyting and others to shape the future of intuitionism.

Did Brouwer's intuitionistic program fail? It certainly did not prevail in the sense that intuitionistic notions did not replace classical ones in everyday mathematics; the antinomies of Cantor's set theory are still present, and undergraduate students encounter intuitionism only in advanced courses. On the other hand, different extensions of intuitionism continued to evolve throughout the years ranging over various disciplines.[4] Heyting's student, Anne Sjerp Troelstra, continued his supervisor's work in formalizing intuitionistic logic and choice sequences, and Michael Dummett developed a philosophical basis for intuitionism by extending Heyting's approach (Troelstra 1969; Dummett 1977). Hermann Weyl's interest in the foundations of mathematics, specifically his *The Continuum,* gave rise to a predicativist point of view which was extended by Solomon Feferman into a mathematical school of its own, namely, Predicativism (Feferman and Hellman 1995; Feferman 2005). More recent developments include semantic interpretations of intuitionism, connections to type theory and computer science, and employment of choice sequences to model indeterminacy in physics (Bezhanishvili and Holliday 2019; Martin-Löf 1984; Gisin 2019).

When discussing the failure of intuitionism to revolutionize mathematics, it's essential to remember two key points. First, failure is not binary; it doesn't imply lack of influence. Second, intuitionism specifically reflects Brouwer's revolutionary views, distinct from later adaptations of his ideas. In philosophical discussions about intuitionism, intuitionism is often considered more broadly, encompassing constructive and philosophical themes rooted in Brouwer's work. In this sense, intuitionism's prosperity in contemporary philosophy has little to do with Brouwer's version of intuitionism and more to do with philosophical developments to later versions of intuitionism.

Therefore, even though Brouwer's intuitionism did not dominate mathematics, some intuitionistic arguments never ceased to be discussed. The debate about the legitimacy of Zermelo's axiom of choice is a good example. In 1904, Ernst Zermelo introduced the axiom of choice to prove Cantor's claim that every set can be well-ordered (Zermelo 1904; Bell 2021). The introduction of the axiom provoked

[3] As documented in a letter from Brouwer to Weyl, 16.2.1928 (van Dalen 2011, 329–30).

[4] In philosophy, for example, intuitionism has a seat at the table in almost every textbook on the philosophy of mathematics. Further analysis of the philosophical community's engagement with Brouwer's intuitionism can be found in Chap. 6.

considerable criticism from the mathematical community, primarily from the French Semi-intuitionists Rene Baire, Emile Borel, and Henri Lebesgue. They argued that the axiom is not constructive as it only asserts the possibility of making several choices but does not indicate how choice functions can be defined (Diaconescu 1975). On the other hand, Hilbert regarded the axiom of choice as an essential principle of mathematics (Moore 1982). Poincaré disagreed with Hilbert, and in 1912 in a lecture on the status of set theory he delivered in Göttingen, Poincaré regarded Zermelo's line of argument as nothing but a word game that lacked all mathematical substance (Gray 2012; Rowe 2018). Brouwer commented on Zermelo's proof in his thesis in 1907, stressing that he agrees with the French semi-intuitionists' critique (Brouwer 1907, 84). The discussion around the axiom of choice eventually led to Brouwer's introduction of choice sequences (Jervell 1996).

However, the echoes of intuitionistic ideas in mainstream mathematics did not end there. Several books that were written after 1930 mention in a footnote or the preface that they are aware of the ongoing debate about the legitimacy of using the axiom of choice and made some adjustments accordingly. Van der Waerden's *Moderne Algebra* and the differences between its three editions regarding the use of the axiom of choice demonstrate the evolution of such changes. One of the major differences between the first edition that came out in 1930 (van der Waerden 1930) and the second edition in 1937 is the complete omission of the parts which rest on the axiom of choice due to van der Waerden's attempt to "avoid as much as possible any questionable set-theoretical reasoning in algebra (van der Waerden 1937, v)." However, in the third edition published in 1951 (van der Waerden 1951), the axiom of choice was reinstated. The decision to include or exclude the axiom of choice suggests that even when not accepted, intuitionistic ideas were not dismissed offhand but carefully reconsidered, at least by some mathematicians, time and again. This low-but-steady interest in Brouwer's intuitionism makes it a rather unconventional but telling case study for theories of framework transitions, especially for the four models examined in this book.

## 1.2 Book Contours

The structure of the book follows a deliberate progression that mirrors the historical and conceptual development of intuitionism within mathematics and philosophy. Each chapter builds on the previous one to explore how individuals and communities can drive or hinder scientific change.

Chapter 2 introduces Brouwer's intuitionism through Leo Corry's model of mathematical frameworks, analyzing the contrasting versions proposed by Brouwer, Hermann Weyl, and Arend Heyting (Corry 1989, 2001). Using Corry's distinction between the "body" and "image" of knowledge, the chapter shows how different conceptions of normative change shaped each approach. I conclude by asking whether philosophical commitments, such as Brouwer's, can motivate mathematical transformation, which leads to the next chapter's focus on the parallel connection between philosophy and science.

Chapter 3 applies Michael Friedman's model of parallel developments to Brouwer's case (Friedman 2001, 2010). Friedman argues that scientific advances are often driven by philosophical breakthroughs, using Einstein's general relativity as an example. Yet Brouwer's philosophically motivated reformation failed to gain traction. I explore this tension by comparing Brouwer's views with those of Kant, the French semi-intuitionists, and Poincaré, showing how his position diverged from each.

Chapter 4 turns to Lukas M. Verburgt's post-Kuhnian historiography, which engages both Corry's and Friedman's accounts (Verburgt 2015). Drawing on Verburgt's distinction between two kinds of *a priori*, I reinterpret Brouwer's intuitionism as a shift between images of knowledge within the same *a priori* framework. I also critically examine Verburgt's reading of intuitionism's influence on "Moscow mathematics," particularly Aleksandr Khinchin, and highlight ambiguities in his concept of epistemic breaks.

Chapter 5 shifts focus to Weyl, a key early supporter of intuitionism. Through Menachem Fisch's model of normative indecision (Fisch 2010, 2017), I explore how Weyl's philosophical ambivalence, especially concerning logical existence and the continuum, emerges from a complex process of self-deliberation and external critique. Comparing his trajectory with George Peacock's hybrid work, I analyze how different forms of conceptual hybridity affect community reception.

While the first four chapters focus on responses from the mathematical community, Chapter 6 examines how intuitionism has been received by philosophers. Through an analysis of major philosophical texts and collections, I show that intuitionism, though marginalized in mathematics, remains influential in philosophical discussions which suggests a divergent narrative of its significance.

Chapter 7 extends the analysis beyond formal structures to interpersonal and institutional dynamics. Through three case studies: (1) Weyl's relocation from Göttingen to Princeton, (2) Brouwer's engagement with the Significs group (Brouwer 1937), and (3) Brouwer's conflict with Hilbert (van Dalen 1990), I illustrate how identity, professional displacement, and interpersonal tensions shape mathematical developments. The final chapter argues that mathematical change is influenced not only by logical and conceptual innovation but also by social, psychological, and institutional factors. By tracing these dimensions, the book offers a broader framework for understanding how robust scientific knowledge emerges through the interplay of individual agency and communal norms.

## References

Bell, John. 2021. The axiom of choice. In *The Stanford Encyclopedia of philosophy*. Stanford University.

Bezhanishvili, Guram, and Wesley H. Holliday. 2019. A semantic hierarchy for intuitionistic logic. *Indagationes Mathematicae* 30 (3): 403–469.

Brouwer, Luitzen Egbertus Jan. 1905. Life, art, and mysticism. *Notre Dame Journal of Formal Logic* 37:381.

Brouwer, Luitzen Egbertus Jan. 1907. *On the foundations of mathematics*. Ph.D. Dissertation, University of Amsterdam.

Brouwer, Luitzen Egbertus Jan. 1911. Über Abbildung von Mannigfaltigkeiten. *Mathematische Annalen* 71 (1): 97–115.

Brouwer, Luitzen Egbertus Jan. 1918. Founding set theory independently of the principle of the excluded middle. *KNAW Verhandelingen* 5:1–43.

Brouwer, Luitzen Egbertus Jan. 1919. Begründung Der Mengenlehre Unabhängig Vom Logischen Satz Vom Ausgeschlossenen Dritten. Zweiter Teil, Theorie Der Punktmengen. *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam* 7:1–33.

Brouwer, Luitzen Egbertus Jan. 1923. Begründung Der Funktionenlehre Unabhängig Vom Logischen Satz Vom Ausgeschlossenen Dritten. Erster Teil, Stetigkeit,Messbarkeit, Derivierbarkeit. *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam* 2:1–24.

Brouwer, Luitzen Egbertus Jan. 1925. Zur Begründung Der Intuitionistischen Mathematik I. *Mathematische Annalen* 93:244–257.

Brouwer, Luitzen Egbertus Jan. 1926. Zur Begründung Der Intuitionistischen Mathematik II. *Mathematische Annalen* 95:453–472.

Brouwer, Luitzen Egbertus Jan. 1927. Zur Begründung Der Intuitionistischen Mathematik III. *Mathematische Annalen* 96:451–488.

Brouwer, Luitzen Egbertus Jan. 1928. Beweis Dass Jede Menge in Einer Individualisierten Menge Enthalten Ist. *Proceedings, Koninklijke Nederlandse Akademie van Wetenschappen. Series C, Biological and Medical Sciences* 31:380–381.

Brouwer, Luitzen Egbertus Jan. 1937. Signific dialogues. In *L. E. J. Brouwer - Collected works Vol. 1 - Philosophy and foundations of mathematics*, 447–452. North-Holland Publishing.

Brouwer, Luitzen Egbertus Jan. 1946. Synopsis of the signific movement in the Netherlands. Prospects of the signific movement. In *Collected works: Philosophy and foundations of mathematics*, ed. Arend Heyting, 465–471. Elsevier Science Publishing.

Brouwer, Luitzen Egbertus Jan, and B. De Loor. 1924. Intuitionistischer Beweis Des Fundamentalsatzes Der Algebra. *Proceedings, Koninklijke Nederlandse Akademie van Wetenschappen. Series C, Biological and Medical Sciences* 27:186–188.

Corry, Leo. 1989. Linearity and reflexivity in the growth of mathematical knowledge. *Science in Context* 3 (2): 409–440.

Corry, Leo. 2001. Mathematical structures from Hilbert to Bourbaki: The evolution of an image of mathematics. In *Changing images in mathematics: From the French revolution to the new millennium*, ed. Umberto Bottazzini and Amy Dahan-Dalmédico, 167–186. Harwood Academic Publishers.

Crowe, Michael J. 1975. Ten laws concerning patterns of change in the history of mathematics. *Historia Mathematica* 2:161–166.

Diaconescu, R. 1975. Axiom of choice and complementation. *Proceedings of the American Mathematical Society* 51:176–178.

Dummett, Michael. 1977. *Elements of intuitionism*. Oxford: Clarendon Press.

Feferman, Solomon. 2005. Predicativity. In *The Oxford handbook of philosophy of mathematics and logic*, ed. Stewart Shapiro. Oxford: Oxford University Press.

Feferman, Solomon, and Geoffrey Hellman. 1995. Predicative foundations of arithmetic. *Journal of Philosophical Logic* 24:1–17.

Ferreira, Ciro, and Cibelle Silva. 2020. The roles of mathematics in the history of science: The mathematization thesis. *Transversal: International Journal for the Historiography of Science* 8.

Fisch, Menachem. 2010. Towards a history and philosophy of scientific agency. *Monist* 93 (4): 518–544.

Fisch, Menachem. 2017. *Creatively undecided: Toward a history and philosophy of scientific agency*. Chicago: The University of Chicago Press.

Friedman, Michael. 2001. *Dynamics of reason*. Stanford, CA: Center for the Study of Language and Information.

Friedman, Michael. 2010. A post-Kuhnian approach to the history and philosophy of science. *The Monist* 93 (4): 497–517.
Gisin, Nicolas. 2019. Indeterminism in physics, classical chaos and bohmian mechanics: Are real numbers really real? *Erkenntnis*. https://doi.org/10.1007/s10670-019-00165-8.
Gray, Jeremy. 2012. *Henri Poincaré: A scientific biography*. Princeton, NJ: Princeton University Press.
Hesseling, Dennis E. 2003. *Gnomes in the fog: The reception of Brouwer's intuitionism in the 1920s*. Basel, Boston, Berlin: Birkhu-ser Verlag.
Heyting, Arend. 1930a. Die Formalen Regeln Der Intuitionistischen Logik I. In *From Brouwer to Hilbert: The debate on the foundations of mathematics in the 1920s*, ed. Paolo Mancosu, 311–327. Oxford: Oxford University Press.
Heyting, Arend. 1930b. Die Formalen Regeln Der Intuitionistischen Logik II. *Sitzungsberichte der Preussischen Akademie der Wissenschaften* 1930:57–71.
Heyting, Arend. 1930c. Die Formalen Regeln Der Intuitionistischen Logik III. *Sitzungsberichte der Preussischen Akademie der Wissenschaften* 1930:158–169.
Heyting, Arend. 1980. *Collected papers*. Amsterdam: North-Holland.
Hilbert, David. 1902. Mathematical problems. *Bulletin of the American Mathematical Society* 8:437–479.
Jervell, Herman. 1996. From the axiom of choice to choice sequences. *Nordic Journal of Philosophical Logic* 1:95–98.
Kirkels, Mireille. 2013. *Mannoury's significphilosophy*. The Netherlands.
Kolmogorov, A. N. 1925. On the principle of excluded middle. In *From Frege to Godel*, ed. Jean van Heijenoort, 414–437. Cambridge, MA: Harvard University Press.
Kuhn, Thomas S. 1962. *The structure of scientific revolutions*. Chicago: University of Chicago Press.
Mancosu, Paolo, ed. 1998. *From Brouwer to Hilbert: The debate on the foundations of mathematics in the 1920s*. Oxford: Oxford University Press.
Martin-Löf, P. 1984. *Intuitionistic type theory*. Naples: Bibliopolis.
McCarty, D. C. 2005. Intuitionism in mathematics. In *The Oxford handbook of philosophy of mathematics and logic*, ed. Stewart Shapiro, 356–386. Oxford University Press.
Moore, Gregory H. 1982. Zermelo's axiom of choice: Its origins, development, and influence. *Studies in the History of Mathematics and Physical Sciences* 8:410.
Posy, Carl. 1998. Brouwer versus Hilbert: 1907-1928. *Science in Context* 11 (2): 291–325.
Putnam, Hilary, and Paul Benacerraf, eds. 1984. *Philosophy of mathematics: Selected readings*. 2nd ed. Cambridge: Cambridge University Press.
Rowe, David. 2018. *A Richer picture of mathematics: The Göttingen tradition and beyond*. Berlin: Springer.
Thurston, William P. 1995. On proof and progress in mathematics. *For the Learning of Mathematics* 15 (1): 29–37.
Troelstra, Anne Sjerp. 1969. *Principles of intuitionism*. Berlin: Springer.
van Atten, Mark. 2004. *On Brouwer*. Belmont: Wadsworth/Thomson Learning.
van Atten, Mark. 2017. The development of intuitionistic logic. In *The Stanford Encyclopedia of philosophy (Winter 2017 Edition)*. Stanford University.
van Atten, Mark. 2020. Luitzen Egbertus Jan Brouwer. In *The Stanford Encyclopedia of philosophy*. https://plato.stanford.edu/archives/win2017/entries/brouwer/. Stanford University.
van Dalen, Dirk. 1990. The war of the frogs and the mice, or the crisis of the mathematische annalen. *The Mathematical Intelligencer* 12 (4): 17–31.
van Dalen, Dirk. 1995. Hermann Weyl's intuitionistic mathematics. *The Bulletin of Symbolic Logic* 1 (2): 145–169.
van Dalen, Dirk, ed. 2011. *The selected correspondence of L.E.J. Brouwer. Sources and studies in the history of mathematics and physical science*. London: Springer.
van Dalen, Dirk. 2013. *L.E.J. Brouwer – Topologist, intuitionist, philosopher: How mathematics is rooted in life*. London: Springer.

van der Waerden, Bartel Leender. 1930. *Moderne Algebra. Unter Benutzung Der Vorlesungen von E. Artin Und E. Noether*. 1st ed. Berlin: Verlag Julius Springer.

van der Waerden, Bartel Leender. 1937. *Moderne Algebra. Unter Benutzung Der Vorlesungen von E. Artin Und E. Noether*. 2nd ed. Berlin: Verlag Julius Springer.

van der Waerden, Bartel Leender. 1951. *Moderne Algebra. Unter Benutzung Der Vorlesungen von E. Artin Und E. Noether*. 3rd ed. Berlin: Verlag Julius Springer.

van Stigt, Walter. 1990. *Studies in the history and philosophy of mathematics: Brouwer's intuitionism*. Amsterdam: North-Holland.

Verburgt, Lukas M. 2015. *"A terrible piece of bad metaphysics"? Towards a history of abstraction in nineteenth and early twentieth-century probability theory, mathematics and logic*. University of Amsterdam.

Weisgerber, Simon. 2023. Mathematical progress — On Maddy and beyond. *Philosophia Mathematica* 31 (1): 1–28. https://doi.org/10.1093/philmat/nkac019.

Weyl, Hermann. 1921. On the new foundational crisis in mathematics. In *From Brouwer to Hilbert: The debate on the foundations of mathematics in the 1920s*, ed. Paolo Mancosu, 86–118. Oxford: Oxford University Press.

Wigderson, Avi. 2019. *Mathematics and computation*. Princeton University Press. https://doi.org/10.2307/j.ctvckq7xb.

Zermelo, Ernst. 1904. Proof that every set can be well ordered. In *From Frege to Gödel: A source book in mathematical logic, 1879–1931*, ed. Jean van Heijenoort, 139–141. Cambridge, MA: Harvard University Press.